2.3 OVERLAND FLOW

Overland flow or sheetflow above land surface in the South Florida Water Management Model involves the movement of surface water either: (a) from cell to cell (internodal flow); or (b) from canal to cell and vice versa. This section focuses on the cell to cell flow process involved in moving surface water from one cell to the next. Section 2.5 discusses the mechanics involved in a canal-to-cell or cell-to-canal overland flow. Succeeding references to overland flow in this documentation refer to cell-to-cell overland flow, unless otherwise noted.

Governing Equations

The diffusion flow model (Akan and Yeh, 1981) is used to simulate overland flow in the South Florida Water Management Model. The primary driving force for diffusion flow is the slope of the water surface. Although a diffusion wave model can account for backwater effects through the pressure terms in the momentum equation, the absence of the inertial or acceleration terms prohibit water from traveling opposite head gradients.

Using water depth as a variable, the two-dimensional continuity equation for shallow water flow is

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} - q = 0 \qquad (2.3.1)$$

where:

h = water depth;
u, v = velocity in the x- and y- directions; and
q = vertical influx which consists of the net effect of rainfall, infiltration and evapotranspiration.

Expressing depth of flow as water level above a datum, the momentum equation in the x-direction can be expressed as

$$\frac{\partial H}{\partial x} + \frac{\tau_{bx}}{\rho g h} = 0 \qquad (2.3.2)$$

while the momentum equation in the y-direction is

$$\frac{\partial H}{\partial y} + \frac{\tau_{by}}{\rho g h} = 0 \qquad (2.3.3)$$

where:

H = h + z = water level above a given datum; The SFWMM uses the National Geodetic Vertical Datum (NGVD) of 1929.;
h = depth of flow; The hydraulic radius is essentially the depth of flow for wide channels, as in the case for SFWMM.;
z = channel bottom elevation above the datum;

τ_{bx}, τ_{by} = bed (bottom and sides) shear stress in the x- and y- directions;
ρ = density of water; and
g = acceleration due to gravity.

The derivation of the above three partial differential equations assumes the following conditions.

1. Vertical velocities and accelerations are neglected, thus flow is essentially two-dimensional.
2. The fluid is incompressible and has uniform density.
3. Bottom slope is small and the channel bed is fixed (no scouring or deposition).
4. Flow is assumed to vary gradually so that hydrostatic pressure prevails.
5. Manning's equation, which applies to steady uniform turbulent flow, can be used to describe bottom resistance effects or bed shear stress, i.e., the slope of the energy grade line S_f can be approximated by means of a semi-empirical formula valid for steady flow.
6. Coriolis effects, surface resistance (wind) stress and shear stresses due to turbulence are ignored.

The bed shear terms can be defined as

$$\vec{\tau}_b = \rho g h \vec{S}_f \tag{2.3.4}$$

where $\vec{\tau}_b$ is the resultant bed shear stress in the direction of the maximum energy slope S_f. Applying Manning's equation in the direction of flow, i.e., direction of maximum energy slope, we have

$$V = \frac{1.49}{n} h^{\frac{2}{3}} S_f^{\frac{1}{2}} \tag{2.3.5}$$

where:

$V = \sqrt{v^2 + u^2}$ = magnitude of the velocity vector;
n = Manning's roughness coefficient; and
$S_f = \sqrt{S_x^2 + S_y^2}$ = magnitude of energy slope.

The direction of flow forms an angle of $\theta = \cos^{-1}\left(\frac{u}{V}\right)$ with the x-axis. Since $\tau_{bx} = \tau_b\cos\theta$ and $\tau_{by} = \tau_b\sin\theta$, and from the preceding two equations, (2.3.4) and (2.3.5), an expression for the two components of bed shear stress can be stated as

$$\tau_{bx} = -\frac{\rho g n^2 u V}{(1.49)^2 h^{\frac{1}{3}}} \tag{2.3.6}$$

$$\tau_{by} = -\frac{\rho g n^2 v V}{(1.49)^2 h^{\frac{1}{3}}} \tag{2.3.7}$$

The negative sign implies that the shear stress goes in the opposite direction to the velocity vector. Based on the third assumption mentioned above, the change in depth of flow with respect to the x or y direction is identical to the change in water level based on the x or y direction. Substituting these equations into the momentum equations yields

$$u = \frac{1.49 h^{\frac{2}{3}} \sqrt{\frac{\partial H}{\partial x}\cos\theta}}{n} \tag{2.3.8}$$

$$v = \frac{1.49 h^{\frac{2}{3}} \sqrt{\frac{\partial H}{\partial y}\sin\theta}}{n} \tag{2.3.9}$$

Since the direction of flow expressed in terms of θ in diffusion flow problems goes in the direction of maximum energy slope, its derivation can be based solely on the slopes of the water surface, i.e., θ can be expressed in terms of H.

Assigning the slope of the water surface to the friction slope in the x- and y- directions:
$S_x \doteq \frac{\partial H}{\partial x}$ and $S_y \doteq \frac{\partial H}{\partial y}$, the maximum energy slope becomes the resultant water surface slope

$S_f \doteq S_n = \sqrt{\left(\frac{\partial H}{\partial x}\right)_x^2 + \left(\frac{\partial H}{\partial y}\right)_y^2}$ and the angle between the flow direction and the x- axis can be

calculated as $\theta \doteq \cos^{-1}\left(\frac{S_x}{S_n}\right)$. Therefore, the u and v velocity components can also be expressed as

$$u = 1.49 \frac{h^{\frac{2}{3}}}{n\sqrt{S_n}} \frac{\partial H}{\partial x} \tag{2.3.10}$$

$$v = 1.49 \frac{h^{\frac{2}{3}}}{n\sqrt{S_n}} \frac{\partial H}{\partial y} \tag{2.3.11}$$

By substituting Eqs. (2.3.10) and (2.3.11) into the continuity equation, (2.3.1), the set of three partial differential equations representing overland flow reduces into a single equation with water level or depth of flow as the only unknown variable.

Model Implementation

The South Florida Water Management Model uses a finite difference approximation of the preceding governing equations to calculate flow velocities in the x- and y- directions, u and v, for each grid cell. The numerical method employed, alternating-direction explicit or ADE, uses the stage values from the previous time step for a particular cell (source cell) and two of its immediate neighboring cells (destination cells): one just to the right (or just to the left) of the source cell, and the other just below (or just above) the same source cell. Two velocities, Eqs. (2.3.10-11), are calculated based on satisfying the diffusion flow model for overland flow. However, violation of the stability condition is avoided by limiting the amount of water across the boundary of two adjacent cells by taking the minimum of (a) the available volume of water from the source cell; (b) flow rate x time; or (c) flow volume required to obtain identical ponding depths between source and destination cells at the end of the time step. The final stage at the source and destination cells are determined by the minimum of the flow volumes resulting from these three conditions. In order to maintain stability and still use the diffusion equation for the majority of the simulation, the model is capable of breaking the standard 1-day time step in the overland flow subroutine into several time slices.

The model uses four six-hour time slices for each day of overland flow calculations. A complete pass of all the grid cells is accomplished for each time slice. Thus, the ponding depth at each cell is updated four times in the course of one day. The difference in the calculations from one time slice to the next is based on the sequence in which source cells are selected and the order in which the two destination cells are selected for each source cell. For the first and third time slices, the left-to-right, top-to-bottom sequence is used. At any given grid cell, the first time slice calculates the flow velocity in the east direction before calculating the flow velocity in the south direction. The order is reversed for the third time slice. In the first and third time slices, the cells immediately to the east and south are referred to as the destination cells and flows across the right and bottom faces of the source cell are calculated. For the second and fourth time slices, the right-to-left, bottom-to-top sequence is used. At any grid cell, the second time slice calculates the flow velocity in the west direction first before calculating the flow velocity in the north direction. The order is reversed for the fourth and final time slice. In the second and fourth time slices, the cells immediately to the west and north are referred to as the destination cells and flows across the left and top faces of the source cells are calculated. Figure 2.3.1 shows the location of computation (source and destination) cells used in the overland flow subroutine of the SFWMM.

The finite difference approximation of Eq. (2.3.10) for the horizontal flow velocity becomes:

$$VOF_x = \frac{1.49}{n} h^{\frac{2}{3}} \left(\frac{\Delta H_x}{\Delta x} \right) \left(\sqrt{ \left(\frac{\Delta H_x}{\Delta x} \right)^2 + \left(\frac{\Delta H_y}{\Delta y} \right)^2 } \right)^{-\frac{1}{2}} \qquad (2.3.12)$$

where:

VOF_x = finite difference approximation of u;
h = water depth at the source cell;
ΔH_x = HS - HD_x = difference in water level between source cell S and destination cell D_x in the x-direction;
ΔH_y = HS - HD_y = difference in water level between source cell S and destination cell D_y in the y-direction;
Δx = horizontal distance between the centers of source cell S and destination cell D_x;

and

Δy = vertical distance between the centers of source cell S and destination cell D_y.

Since the model only handles square cells ($\Delta L = \Delta x = \Delta y$), hence

$$VOF_x = \frac{1.49}{n} h^{\frac{2}{3}} \left(\frac{\Delta H_x}{\Delta L} \right) \left(\sqrt{\frac{\Delta H_x^2 + \Delta H_y^2}{\Delta L^2}} \right)^{-\frac{1}{2}}$$

which can be rearranged to

$$VOF_x = \frac{1.49}{n} h^{\frac{2}{3}} \left(\frac{\Delta H_x}{\Delta L^{\frac{1}{2}}} \right) \left(\frac{1}{\sqrt{\Delta H_x^2 + \Delta H_y^2}} \right)^{\frac{1}{2}} \qquad (2.3.13)$$

From Eq. (2.3.13), the flow rate in the x-direction can be calculated as

$$Q_x = VOF_x \bullet h \bullet WDTHOV_x \qquad (2.3.14)$$

where $WDTHOV_x$ is the width of overland flow in the x-direction (Δy).

The solution to the diffusion equation will yield a volume of overland flow in the x-direction

$$VOLOV_x = Q_x \bullet DTS \qquad (2.3.15)$$

where DTS equals the length of a time slice. To maintain stability, the model limits this volume by the two other parameters discussed earlier. The limiting flow volume becomes the basis for the final stages at the three computational cells. The corresponding flow velocity, flow rate and volume of overland flow in the y-direction are:

$$VOF_y = \frac{1.49}{n} h^{\frac{2}{3}} \left(\frac{\Delta H_y}{\Delta L^{\frac{1}{2}}} \right) \left(\frac{1}{\sqrt{\Delta H_x^2 + \Delta H_y^2}} \right)^{\frac{1}{2}} \qquad (2.3.16)$$

$$Q_y = VOF_y \bullet h \bullet WDTHOV_y \qquad (2.3.17)$$

$$VOLOV_y = Q_y \bullet DTS \qquad (2.3.18)$$

Figure 2.3.1 shows a typical computational grid used in the overland flow subroutine. The head at the grid cell denote by (i,j) at time step t+1 is a function of the head at three adjacent cells evaluated at the previous time step t. The selection of which two cells, in addition to itself, to consider depends on the current time slice, as mentioned earlier.

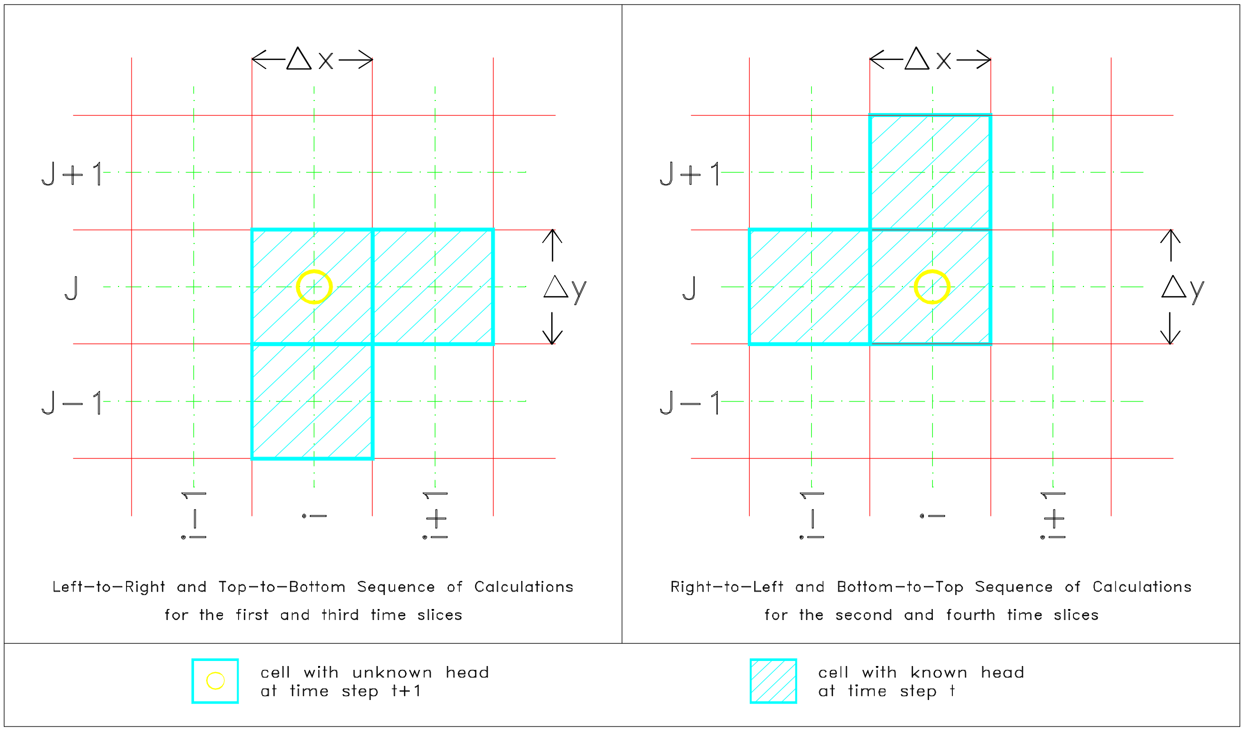

Figure 2.3.1 Location of Computation (Source and Destination) Cells Used in Calculating Total Head at Grid Cell (i,j) During Time Step t+1 as Implemented in the Overland Flow Subroutine in the South Florida Water Management Model

Resistance to Sheetflow

Movement of water above land surface, sheetflow or overland flow, is governed by two sets of parameters in the model: detention depth and roughness. Detention depth (DETEN) is the depth of ponding within a grid cell below which no transfer of water from one grid cell to the next is allowed even if a hydraulic gradient exists between the adjacent cells. Detention depth is used to characterize water retained as puddles in small surface depressions that may sporadically exist at varying sizes within a 2-mile by 2-mile model grid cell. The model treats each grid cell as a perfectly horizontal surface. If detention depth is exceeded and a gradient is established between adjacent cells, surface roughness determines the magnitude of flow. The roughness parameter used is Manning's roughness coefficient *n* or simply Manning's *n*. Lower *n* values, i.e., less resistance to flow, are associated with larger ponding depths due to the flattening of vegetation as flow velocities increase. In the model, Manning's *n* is an exponential function of ponding depth (POND):

$$n \;=\; A \bullet \mathrm{POND}^{b} \tag{2.3.19}$$

It is computed for each grid cell every time step based on land use and ponding depth at the grid cell. Table 2.3.1 shows the coefficients A and b used to determine Manning's *n* for the fourteen land use types in the model.

Table 2.3.1 Overland Flow Resistance Coefficients Used in the SFWMM (cell-to-cell overland flow)

	Manning's *n*		Detention Depth
Land Use Type	**A**	**b**	**DETEN, ft**
LU1: Low-density Urban	0.200	0.00	0.08
LU2: Agricultural	0.225	0.00	0.08
LU3: Fresh Marsh	1.300	-0.770	0.08
LU4: Sawgrass	1.050	-0.770	0.08
LU5: Wet Prairie	0.365	-0.770	0.08
LU6: Scrub and Shrub	0.760	-0.770	0.07
LU7: Truck Crops	0.225	0.00	0.08
LU8: Sugar Cane	0.225	0.00	0.07
LU9: Irrigated Pasture	0.225	0.00	0.07
LU10: STA Wetland	1.300	-0.770	0.058
LU11: High-density Urban	0.080	0.00	0.10
LU12: Forest	0.550	0.00	0.06
LU13: Mangrove	1.350	-0.770	0.08
LU14: Melaleuca	0.750	0.00	0.10

note: Land use types LU7, LU8 and LU9 are the three predominant land use classifications in the EAA. However, since overland flow is not simulated in the EAA (refer to Sec. 3.3), the coefficients corresponding to these land use types are not used in the model.